Soft random geometric graphs

Mathew Penrose (University of Bath, UK)

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The soft random geometric graph (SRGG) in 2 dimensions is obtained by placing n vertices uniformly at random in $[0,1]^2$ and connecting any two vertices distant at most r apart with probability p. We discuss large-n asymptotics with $r = r_n$ a specified sequence and $p \in (0,1)$ a fixed parameter.

Let L_n denote the order of the largest component of this graph.

In the 'thermodynamic limit' where $nr_n^2 \to c \in (0,\infty)$ we discuss a law of large numbers for L_n, namely L_n/n converges in probability to a constant (Penrose 2022).

In the 'connectivity regime' where $np\pi r_n^2 - \log n$ tends to real-valued limit β, the probability that the graph is fully connected tends to $\exp(-e^{-\beta})$. (Penrose 2016) and we discuss this.

The SRGG generalizes both the 'hard' random geometric graph (take $p = 1$) and the classical Erdos-Renyi random graph (take $r = \sqrt{2}$ and allow p to vary with n). Results analogous to both of those described above are available for both of these models (with different proofs); however, further ideas were required for the proofs for the SRGG.

[1] Penrose, M. D. (2016) Connectivity of soft random geometric graphs. *Ann. Appl. Probab.* **26**, 986–1028.

[2] Penrose, M. D. (2022) Giant component of the soft random geometric graph.(English summary) *Electron. Commun. Probab.* **27**, Paper No. 53, 10 pp.