Scattering problems and asymptotic models at a fixed frequency

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January 14-18, 2008

The solutions to scattering problems in harmonic regime contain an intrinsic length of oscillation λ, called the wavelength. Any numerical method based a discretization of the computational domain should therefore use a mesh size fine enough to capture the variations of the solutions within a period of space λ. Many practical problems require however a stronger constrain on the mesh size due to the presence of a smaller scale δ induced by geometrical fine details or by physical strong contrasts. The cases where $\delta \ll \lambda$ therefore lead to a significant increase in the size of the discrete model and consequently the cost of the numerical scheme.

This course will provide an introduction to some asymptotic techniques that can be used to replace the exact model with approximate ones whose numerical resolution does not suffer from that stronger constrain and whose solution will converge to the exact one as δ goes to zero. In general, one can construct a hierarchy of models with increasing rate of convergence in terms of δ, but also with increasing complexity.

The main focus of this course will be on the theoretical side: understanding the principle of asymptotic expansions, formally deriving approximate models and obtaining error estimates through stability analysis of the problems with respect to the small parameter.

After a general introduction to scattering problems in harmonic regime (we shall restrict ourselves to the scalar problem modeled by the Helmholtz equation) we shall investigate three typical asymptotic configurations associated with:

1. The scattering from coated obstacles of thickness $\delta \ll \lambda$.

2. The scattering from strongly absorbing obstacles where the small scale δ is produced by the boundary layer effect due to strong absorption.

3. The scattering from periodic media where the periodicity scale δ is very small compared to λ.